Universidad Rey Juan Carlos	**Physics** **Biomedical Engineering** **Course 2020 - 2021**	**Problem Sheet 1** **Dimensions and Vectors**

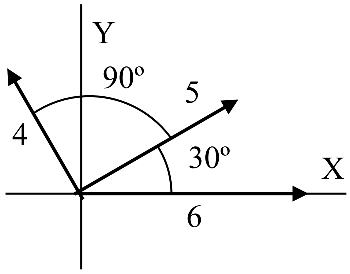
1. Consider the equation $v = \frac{1}{3} \cdot zxt^2$. The dimensions of variables x, v and t are $[x] = L$, $[v] = LT^{-1}$ and $[t] = T$. Find the dimensions of z variable for making the equation self-consistent? **Ans**: $[z] = T^{-3}$

2. An object is linearly moving with an acceleration $a(t) = a_0\, t + a_1\, e^{\gamma t} + a_2\, sen(\omega\, t)$. Determine the dimensions for a_0, a_1, a_2, γ and ω. **Ans.**: $[a_0] = LT^{-3}$; $[a_1] = [a_2] = LT^{-2}$; $[\gamma] = [\omega] = T^{-1}$.

3. Given the following expressions $[a] = L/T^2$, $[v] = L/T$, $[x] = L$ and $[t] = T$, Find the one with the wrong dimensions a) $v^2 = 2ax$, b) $v = at$; c) $v = \frac{x}{t} + at^2$; d) $x = \frac{v^2}{a}$ **Ans.**: c

4. Given the vectors in the figure, Find:

a) Their geometric addition.
b) The components of each vector in the given reference frame.
c) The components of the sum vector.
d) The angle between the sum vector and the largest vector.



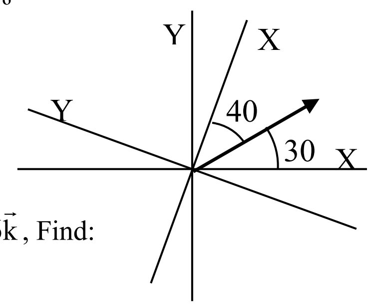
Ans.: b) $(6,0)$, $(5\sqrt{3}/2, 5/2)$, $(-2, 2\sqrt{3})$;c) $(4+5\frac{\sqrt{3}}{2}, \frac{5}{2}+2\sqrt{3})$; d) 35,56º

5. Given the points P (-1, 0, 2) and Q (2, -3, - 5), Find: a) the vector $\vec{r} = \overrightarrow{QP}$; b) the unit vector parallel to $\vec{u}_r$; c) the angle between vector $\vec{r}$ and each coordinate axis.

Ans: a) $\vec{r} = (-3,3,7)$; b) $\vec{u}_r = (-\frac{3}{\sqrt{67}}; \frac{3}{\sqrt{67}}; \frac{7}{\sqrt{67}})$ c) $\gamma_x = 111{,}5º$; $\gamma_y = 68{,}5º$; $\gamma_z = 31{,}1º$

6. The vector **a** in the figure has length of 10 units. Determine the coordinates of this vector: a) respect to *XY axes;* b) respect to *X'Y' axes.*

Ans: a) $a_x = 5\sqrt{3}$, $a_y = 5$; b) $a_{x'} = 7{,}7$; $a_{y'} = 6{,}4$.



7. Given the following vectors: $\vec{a} = -\vec{i} + \vec{j} + 4\vec{k}$, $\vec{b} = -3\vec{i} + \vec{j} - 7\vec{k}$ and $\vec{c} = 4\vec{i} + 7\vec{j} + 6\vec{k}$, Find:

a) $\vec{a} + \vec{b}$, $\vec{c} - \vec{a}$, $\vec{a} \cdot \vec{b}$, $\vec{a} \times \vec{b}$, $\vec{a} \cdot (\vec{b} \times \vec{c})$

b) The angle between a and b.

Ans: a) (-4, 2,-3); (5, 6, 2); -24; (-11,-19, 2); -165; b) 137,43º

8. Given the vector $\vec{a} = 5t^2 \cdot \vec{i} + t \cdot \vec{j} - t^3 \cdot \vec{k}$, Find $\frac{d\vec{a}}{dt}$ and $\int_1^2 \vec{a}\, dt$

Ans: $\frac{d\vec{a}}{dt} = 10t \cdot \vec{i} + \vec{j} - 3t^2 \cdot \vec{k}$; $\int_1^2 \vec{a}\, dt = \frac{35}{3}\vec{i} + \frac{3}{2}\vec{j} - \frac{15}{4}\vec{k}$

9. Find a unit vector on the plane OYZ and perpendicular to vector $\vec{v} = 2\vec{i} + \vec{j} - 3\vec{k}$ **Ans.**: $(3\vec{j} + \vec{k})/\sqrt{10}$

10. Determine and Justify if each of the following propositions is true or false:

a) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the vectors $\vec{a}$ and $\vec{b}$ are perpendicular.

b) Given $\vec{s} = \vec{a} + \vec{b}$ and $\vec{d} = \vec{a} - \vec{b}$, if $\vec{s}$ and $\vec{d}$ are perpendicular to each other, then $|\vec{a}| = |\vec{b}|$

c) If the modulus of vector $\vec{u}(t)$ is constant with time, then $\vec{u}(t)$ and $\frac{d\vec{u}}{dt}$ are perpendicular to each other.

d) If the modulus of the cross product of two vectors is equal to the value of their dot product, the angle between both vectors is 30º.

Ans.: a, b and c true, d false.